WHAT COUNTS AS NUMERACY

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SYNOPSIS

The purpose of the study was to infer the Scottish HMI view of what is meant by Numeracy given the concerns that primary children's achievements in Numeracy reflect a lack of flexibility in handling number and an overemphasis on procedures at the expense of understanding (HMI, 1997). Three hundred HMI reports on primary schools in Scotland were randomly selected. Content analysis of the sections on Number, Money and Measurement revealed Numeracy to be conceived of as computational proficiency and as understanding of number. Surprisingly, there were significantly more ($p<0.05$) references to computational proficiency than there were to understanding of number. The results are discussed in terms of what it means to understand number. It is suggested that there needs to be much clearer delineation of what is required and meant by the idea of understanding number.

INTRODUCTION

The Number Curriculum has long since been concerned with proficient computation (McIntosh, 1981; Brown, 1999; 2001). While computational proficiency is perfectly appropriate, Skemp (1971) points out that what many pupils experience is the rote manipulation of symbols without the opportunity to construct meaning for these symbols. As well as being boring to execute, the rote enactment of meaningless procedures does not of itself allow the pupil to develop an elaborated and integrated cognitive structure for the scope of, and the inter-relationship between, different sets of numbers (that is natural numbers, integers, rational numbers and real numbers). Without such a conceptual structure, the pupil is powerless either to act strategically or solve problems. What this means in practice is that pupils may perform basic algorithmic operations accurately but when these operations are applied to realistic situations (Robertson, Meehan, Clarke and Moffat, 1996) or when the numbers involved include fractions (Wearne and Hiebert, 1988) accurate performance drops. That pupils have difficulty in connecting the procedural and conceptual aspects of number (Hart, 1989) does not mean that they are incapable of number conceptualisation. In the real world children can perform numerical calculations and can reason numerically without using school-taught algorithms (Lave, 1988; Nunes and Bryant, 1996). This disjunction between procedural and conceptual knowledge has been recognised by many, including Her Majesty's Inspectorate (1997), and is typically attributed to pedagogical practices which emphasise symbol manipulation at the expense of conceptual learning (Treffers, 1991; Mayer and Hegarty, 1996).

While, over the years, there has been much pedagogical guidance on how to improve children's Numeracy (Brown, 1999), it is important to remember that learning to implement new teaching strategies or use new materials is only one aspect of changing teachers' practices. A second, and equally important, aspect is the change in teachers' beliefs since beliefs, however implicit or idiosyncratic they may be, strongly affect behaviour (Pajares, 1992). Teachers' ideas about what Numeracy is and about how Numeracy teaching is best effected are closely linked (Stein, Baxter and Leinhardt, 1990). If teachers are to change their practices with the intention of improving children's Numeracy they may also have to change some of their educational beliefs about what constitutes Numeracy and what constitutes the effective learning and teaching of Numeracy. Without such a change in beliefs, teachers' practices can become a pastiche of techniques, strategies, and materials which may be based on different, and even inconsistent, theoretical positions about learning (Schifter and Simon, 1992) with the possible consequence that effective teaching and learning cannot occur because, while surface practices may have been altered, there is no examination by the teachers as to what fundamental purposes are served by changes in the practice (Alexander, 1992). The influence of beliefs is thought to be the variable which explains the difference between effective and less effective teachers of Numeracy when their teaching styles and strategies were observed to be similar (Askew, 1999). Askew, Brown, Rhodes, William and Johnson (1997) designated highly effective teachers as having a connectionist orientation to teaching. A primary belief among teachers with a connectionist orientation is that teaching is based on dialogue between learner and teacher so that teachers better understand learners' thinking and so that learners can access the teacher's knowledge. By way of comparison, Askew *et al* (1997) also inferred a transmission orientation (where the teacher's beliefs focus heavily on the role of the teacher) and a discovery orientation (where the teacher's beliefs focus on children learning independently) to the teaching of Numeracy. While Askew (1999) regards these orientations as ideal types rather than as mutually exclusive categories since teachers are unlikely to have a set of beliefs which precisely matches those delineated within each orientation, he nevertheless argues that teachers are predisposed to talk and behave in ways which accord more with one orientation than with the others.

While beliefs do not translate simply into observable behaviour (Pajares, 1992), there is nevertheless a congruence between teachers' beliefs and their pedagogical practices (Stipek, Girvin, Salmon and MacGyvers, 2001) which reflects the distinction between procedural and conceptual knowledge. Teachers who emphasise pupil performance through getting correct answers, getting high scores and completing tasks speedily, are of the view that ability is stable (Dweck, 2000) and that teachers should exercise complete control over pupil learning and activity (because teachers have a responsibility to transmit to pupils the rules and procedures which yield correct answers). On the other hand teachers who encourage pupil learning through problems/activities that require understanding and reasoning, are of the view that ability develops as a consequence of effort and learning (Dweck, 2000) and that there should be pupil autonomy in the choice and conduct of mathematical activities (Stipek *et al*, 2001).

Given the fundamental role of beliefs in conceptualising and delivering the mathematics curriculum and given the concerns of Her Majesty's Inspectorate (1997) that primary children's achievements in Numeracy reflect a lack of flexibility in handling number and an overemphasis on procedures at the expense of understanding, it seems appropriate to consider what the 'official' conception of Numeracy might be, in Scotland. The Inspectorate is charged with the practical task of inspecting individual schools and reporting on standards. In order to do this, the Inspectorate must have a view on what constitutes Numeracy. Whatever this view might be, it is nevertheless important since the 'official' published pronouncements as to what constitutes effective teaching and learning of Numeracy feed into other reports, initiatives and publications (HMI, 1997).

The Inspectorate's conception of Numeracy could reflect one of two positions. It could be of the view that Numeracy is procedural knowledge which is developed through learning facts and routines and learning how to use these quickly and efficiently. Or it could be of the view that Numeracy is conceptual knowledge which is developed through reasoning about, reflecting upon and arguing about solutions to problems. That these two conceptions are presented as alternatives is perhaps slightly misleading if not erroneous since an emphasis on understanding need not imply a laissez-faire attitude towards computational proficiency. It should not be necessary to sacrifice procedural knowledge for conceptual knowledge, nor conceptual knowledge for procedural knowledge. Both should develop together (McIntosh, Reys and Reys, 1992). While both conceptual and procedural knowledge should develop together, it is nevertheless important to make explicit that the study reported here was conducted on the assumption that being numerate means understanding the concept of number itself, understanding the effects of operations on numbers and understanding the application of numbers and operations to computational settings (McIntosh *et al*, 1992). To the extent that 'understanding' is not emphasised as an integral part of computational proficiency, the dichotomy between conceptual and procedural knowledge still exists.

Because of role of the Inspectorate in shaping the nature and delivery of the curriculum, it was reasoned that their reports on the inspection of schools would be fertile sources of data. Information from the reports was gathered to try to ascertain:

What conception of Numeracy can be inferred from published HMI Reports on school inspection?

METHOD

Three hundred HMI reports on primary schools in Scotland (published between 1988 and 1999) were randomly selected. The time span is the twelve years since the inception of the Curriculum and Assessment 5–14 Programme in Scotland.

Design

In each report the section(s) on mathematics, teaching and learning were identified and, within that, the references to Number, Money and Measurement, an attainment outcome which "is concerned with knowledge and understanding of number and its applications" (SOED, 1991). The sections of text referring to Number, Money and Measurement in each report were the sampling units. The sampling units were not all of equal size, ranging from 48 to 229 words, with the average size being 85 words. This is partially explained by differences in the reports themselves (so that Extended Reports were lengthier than Standard Reports) and partially explained by differences in reporting 'style' (some authors wrote more or less than others on Number, Money and Measurement). The recording units were those segments of text that referred to computational proficiency or to understanding of number. Because, however, the meaning of the text segment is derived in part from the environment in which it occurs, the context unit for determining the category into which the text segment was placed was the grammatical clause or sentence. So, for example, the segment, "multiplication tables" would be coded as referring to computational proficiency in the context, "most senior pupils had good recall of multiplication tables" but would be coded as understanding of number in the context, "pupils could use their multiplication tables when serving in the shop". In a few instances, however, the context was insufficiently explicit to allow a clear categorisation of the text segment. Being "quick at mental tasks" was, for example, ambiguous (and hence categorised as such) in that it is unclear whether this segment refers only to the recall of factual numerical information or whether it refers to strategic numerical thinking and behaviour. It is important to point out that only those text segments which made explicit reference to computational proficiency or understanding of number (as defined below) were counted. Segments which made general reference to learning and teaching and general references to mathematics and number such as "emphasis on number at the expense of some other aspects of maths" or "most pupils attaining the appropriate national targets in number" were not included. It is recognised that the selection of only semantically appropriate segments leaves the study open to the potential criticism that not all of the data were accounted for (Robson, 1993). However, the purpose of the study was not to analyse audit reports *per se* but to interrogate the reports for what they had to say specifically on Numeracy.

Coding Categories

The category definitions were devised by the author but were influenced by the literature which draws attention to a distinction between conceptual and procedural knowledge in the learning of Numeracy (McIntosh *et al*, 1992; Rittle-Johnson and Siegler, 1998; Anghileri, 2000). Segments which were coded as computational proficiency were those that made reference to:

1. number 'facts': that is the quick, accurate and relatively effortless retrieval of the sums or products of integer-pairs, such as 4 plus 5 or 3 times 9;
2. the written algorithms for addition, subtraction, multiplication and division: examples might be the verbal protocols to observe when using either the 'equal additions' or 'decomposition' method for written subtraction;
3. the sequences of actions to be invoked for solving problems: as in activating the mnemonic, RACE CAR (McDougall and Cook, 1993).

Segments which were coded as understanding of number were those that drew attention to the "logical structure underlying numbers and number operations" (Anghileri, 2000). This understanding is principled knowledge which underpins the flexible use of quantitative methods. It refers to the relationships between numerical ideas such that the individual knows:

1. about the concept of number: for example that the digit '2' in 243 is not the same as the digit '2' in 26; or that 3.5 is a rational number because it is the ratio of 35 to 10;
2. the effects of operations on numbers: for example that multiplication may mean repeated addition, but it also might mean rectangular arrays or even scaling; and similarly that division can be understood as sharing, but it can also be understood as repeated subtraction and ratio;
3. that the application of numbers and operations to computational settings means selecting and even creating appropriate strategies depending upon how exact or approximate the data are/or need to be (McIntosh *et al*, 1992).

Procedure

Coding was tested on a sample of reports. The researcher and an assistant who was familiar with the literature on the distinctions between the conceptual and procedural aspects of numeracy independently coded a sample of 50 reports. This testing indicated that the distinction between 'conceptual' and 'procedural' was robust (Kappa Coefficient 0.79) although there was initial disagreement as to how to quantify the segments within each category. For example, in the statement, "most could round numbers to the nearest unit and some could round decimals" both coders agreed that this statement implied an understanding of number and so categorised it as such. The issue turned on whether this statement contained two segments, "could round decimals" and "could round numbers to the nearest unit" or one segment (in which case "could round decimals" would be subsumed by "could round numbers to the nearest unit"). This difference was resolved by agreeing that although conceptually one idea may have been an elaboration of a previous idea, it was nevertheless the report author's intention to *add* information to what had already been said. In the above example there would thus be two segments. Having determined the reproducibility of the coding procedure, the researcher coded the full set of 300 cases twice, with an interval of several weeks between coding and re-coding. The stability of the coding procedure was acceptable with kappa coefficients for procedural knowledge being 0.88 and for conceptual knowledge being 0.78.

RESULTS

Table 1: Number of Reports Sampled 1988–1999

Year	Frequency	%
1988	29	9.7
1989	26	8.7
1990	19	6.3
1991	20	6.7
1992	8	2.7
1993	30	10.0
1994	26	8.7
1995	32	10.7
1996	13	4.3
1997	34	11.3
1998	45	15.0
1999	18	6.0
Total	**300**	**100**

Table 2: Reports Sampled in Grouped Years 1988-1999

Grouped Year	Frequency	%
1988–1990	74	24.7
1991–1993	58	19.3
1994–1996	71	23.7
1997–1999	97	32.3
Total	**300**	**100.0**

Differences in the numbers of reports sampled for each group of years are not significant.

As shown in Table 3, the incidence of ambiguous segments was small. In 256 of the reports there were no ambiguous segments. In 35 of the reports there was one ambiguous segment. In 8 of the reports there were two ambiguous segments and in 1 of the reports there were three ambiguous segments.

One hundred and twelve (112) reports contained no segments referring to Understanding while 1 report contained thirteen segments referring to Understanding. There were many more segments referring to Proficiency. Sixty (60) reports contained no segments referring to Proficiency while 3 of the reports contained fifteen segments referring to Proficiency.

The difference between segments deemed to signify Understanding and segments deemed to signify Proficiency was significant ($Z = -7.23$, $p<0.05$).

Table 3: Frequency of References to Key Components of Numeracy

No.of Segments	Under-standing	Running Total	Proficiency	Running Total	Ambiguous Segments	Running Total
0	112	0	60	0	256	0
1	53	53	53	53	35	35
2	48	96	55	110	8	16
3	26	78	38	114	1	3
4	24	96	26	104	0	0
5	11	55	9	45	0	0
6	9	54	22	132	0	0
7	6	42	9	63	0	0
8	1	8	11	88	0	0
9	4	36	5	45	0	0
10	2	20	4	40	0	0
11	3	33	3	33	0	0
12	0	0	1	12	0	0
13	1	13	0	0	0	0
14	0	0	1	14	0	0
15	0	0	3	45	0	0
Total	**300**	**584**	**300**	**898**	**300**	**54**

Table 4: References to Key Components of Numeracy across Grouped Years

	Number of References			
Time Span	**0**	**1**	**>1**	**Total**
1988–1990				
Understanding	28	25	21	74
Proficiency	16	22	36	74
Ambiguous	65	9	0	74
Sub-total	**109**	**56**	**57**	
1991-1993				
Understanding	5	8	45	58
Proficiency	2	5	51	58
Ambiguous	48	4	6	58
Sub-total	**55**	**17**	**102**	
1994–1996				
Understanding	13	5	53	71
Proficiency	6	6	59	71
Ambiguous	58	12	1	71
Sub-total	**77**	**23**	**113**	
1997–1999				
Understanding	66	15	16	97
Proficiency	36	20	41	97
Ambiguous	85	10	2	97
Sub-total	**187**	**45**	**59**	
Total	**428**	**141**	**301**	**900**

Segments deemed to signify Understanding were significantly different ($\chi^2 = 531.05$, $p<0.05$) across the groups of years. Segments deemed to signify Proficiency were significantly different ($\chi^2 = 319.1$, $p<0.05$) across the groups of years.

DISCUSSION

That there were no significant differences in the numbers of reports analysed in each of the four groups of years suggests that the sample of reports was drawn from an evenly distributed population. This is reassuring since the other analyses reveal instability, which is presumably not a function of faulty sampling. The segments signifying proficiency were significantly more numerous than those signifying understanding. Segments such as "pupils were insecure in their recall of number bonds", "basic number operations were well developed at all stages", "some pupils require to improve their recall of multiplication tables" and "pupils were competent in adding, subtracting, multiplying and dividing numbers at appropriate levels of difficulty" were very plentiful while segments such as "pupils successfully applied their number skills in context", "P5 had difficulty in giving change from £1" and "pupils were confident in equivalence of fractions" were significantly fewer. This might, simplistically, suggest that the Inspectorate conceptualised Numeracy as computational proficiency rather than as understanding. However, that segments signifying proficiency and segments signifying understanding were not evenly distributed rather suggests that the Inspectorate's conception of Numeracy was unstable. Had the Inspectorate's conception been stable, it would have been reasonable to expect the proportions of understanding segments and proficiency segments either to be constant or to follow some trend. The statistics, however, support the contention that the conception of Numeracy purveyed by the Inspectorate is unstable. Why should this be?

Given the concerns of the concerns of Her Majesty's Inspectors (1997) it seems absurd to suggest that they were less concerned with the understanding of number and more concerned with proficiency in number (which is what the statistics suggest). Perhaps however the difficulty hinges on what is meant by understanding number. Understanding is a complex idea with several distinct nuances of meaning (Putnam, Lampert and Peterson, 1990). One meaning of understanding is as connections between different types of knowledge (and such a view of understanding would seem to be consistent with the Inspectorate's concerns). These connections allow a person to operate flexibly with numbers such that, for example, the person uses known number facts to derive facts of which he/she is less sure, decides whether a particular number is a reasonable answer, finds an approximate rather than an exact answer, decomposes and recomposes numbers to simplify a calculation, appreciates the relative size of numbers, is able to move flexibly among different possible representations of numbers (Resnick, 1989b). This flexibility means that the person makes connections between formal and informal knowledge, between conceptual and procedural knowledge (Putnam *et al*, 1990) and between the various pieces of knowledge and skill which the individual has developed (Lesh, Post and Behr, 1987). But these connections are not unproblematic.

Firstly, the connection between formal and informal will not happen without the careful and systematic intervention of the teacher. Many three and four-year olds and most five-year olds (Resnick, 1989a) have developed a rich store of conceptual knowledge about Number. They can count with a tolerable level competence and they can add and subtract (Gelman and Gallistel, 1978; Fuson and Hall, 1983). However the young child's conceptual grasp of Number is at an informal level only and the significance of this informal knowledge for what happens in school need not be obvious to the child (Bryant, 1997). But it would be reasonable to assume that Her Majesty's Inspectors would appreciate the significance of the young child's informal knowledge given the dominant perspective of constructivism (Putman *et al*, 1990) in learning about Number. Unfortunately there was little evidence of this. Many of the reports contained segments to the effect that most pupils "had made a confident start with number" or that P1 pupils "could confidently work with numbers to 10" or that "pupils at the early stages made satisfactory progress in counting and numerical calculations involving addition and subtraction". This should not be surprising since the research evidence (Gelman and Gallistel, 1978; Fuson and Hall, 1983; Resnick, 1989a) suggests that most children do make a very satisfactory start on number knowledge before coming to school. But none of the reports made any reference to helping children to make connections between their own informal knowledge and the formalisms of school. Yes, the Inspectors recognised the need for children to be proficient in formal Number, in segments such as "by P3 half could confidently perform written addition and subtraction" but there was never any attempt to reflect that children can use a range of different calculation strategies to perform (informal) addition and subtraction and that it is the linking of children's own strategies with formal notation which enables the child to impute meaning into the formalisms of Number (Resnick, 1989b). In that there was scant regard for the potential of informal knowledge to illuminate formal Number (a finding that is corroborated in Resnick, 1989a; Brown, 2001), the Inspectors did not recognise the power or significance of children's informal knowledge in the development formal curricular Number knowledge. To this extent the 'official' conception of Numeracy does not include connections between informal and formal knowledge.

Secondly the connection between conceptual and procedural knowledge is difficult to make because conceptual knowledge about number is abstract knowledge. Because number is the property of a set or collection of items rather than the property of the individual items within the set, there are, strictly speaking no denotable objects. So, for example, when we speak of a set of blue plates, the adjective 'blue' describes the plates but when we speak of a set of five plates, the adjective 'five' describes the set, not the plates. From the beginning, then, children have to reason about objects that exist only as mental abstractions. But according to a Piagetian model of development, young children, and this may include many children at primary school (Resnick, 1989a), can reason about Number only when tangible objects and events are present. That concrete operations refer to reasoning which has not yet separated from its empirical context has resulted in teaching practices which typically involve the use of manipulative or concrete materials on the assumption that the experience of concrete materials will enable the child to construct a mental representation of the concept (Orton and Frobisher, 1996). In the present study the importance attributed to manipulatives was evidenced in segments such as "practical materials were used to explore new ideas", "pupils were able to use structured materials for place value", "a practical approach is also used in introducing simple fractions", "more practical experiences should be provided to enable pupils to learn and understand basic number relationships". Almost always, references to concrete materials implied that their use was a 'good thing' as in "apparatus to aid number work should be used more extensively". The exception to this was the comment, a version of which appeared more than once, that "too many children were over-reliant on finger counting". That concrete materials were viewed so positively while the use of fingers was viewed negatively suggests that fingers were not seen as concrete materials!

However, the use of concrete materials has not necessarily produced the intended outcome of meaningful links between procedural and conceptual knowledge (Hart, 1989; Boulton-Lewis, 1993; Boulton-Lewis and Tait, 1994; Hall, 1998). The reasons for this are complex. One reason posited is that since numerical meaning does not reside in the materials but has to be imposed on them the materials can only mediate understanding if the structure of the numerical idea is recognised by the child (Cobb, 1987). In other words the concrete materials will only be meaningful to those children who already have the concept which the materials are supposed to exemplify. One implication of this is that the materials themselves may be potential barriers to child's developing conceptualisation of number. Another reason posited for the failure of concrete materials to promote understanding is that the use of materials is imposed, or at least legitimised, by the teacher (as seems to be the case in this study) and therefore may not articulate with the informal knowledge and strategies of the child (Gravemeijer, 1997). Although the role of concrete materials in the development of understanding would appear to be more complex than was originally understood (Lesh *et al*, 1987), the assumption that concrete materials will as a matter of course help children to develop number concepts and skills in a more meaningful way is clear in the Inspectors' pronouncements in the audit reports. To the extent that the role of concrete materials is questionable, the 'official' conception of Numeracy is limited by what seems to be a lack of awareness of research which surrounds the issue of Number representation.

Thirdly, to suggest that understanding is about making connections between different types and pieces of knowledge means that the learner can use extant knowledge to interpret and structure new knowledge. That learners build new knowledge on the basis of previous knowledge (and, by implication, that each learner brings to any situation a unique web of knowledge) means that prior or extant knowledge is critically important. But what is also important is the nature of the relationship between prior knowledge and new knowledge. As was argued earlier, the young child's informal knowledge is an essential precursor to the development of meaningful formal work on Numeracy. Most children come to school with the idea that numbers are grounded in the counting principles and the related activities of addition and subtraction (Gelman & Gallistel, 1978). This knowledge serves children in the early primary school well because the formal tasks in which they are expected to engage depend almost exclusively on the concept of natural number (cf. SOED, 1991). In the early years only cursory reference is made to fractions and not until Level E is there any reference to negative numbers. This is not to be critical of the learning targets to be achieved, but merely to emphasise that the mathematical understanding required in most of the formal tasks in early primary is that numbers are sets of things which can be enumerated.

However, what children's informal knowledge can contribute to the formal learning of 'topics' such as multiplication, division and fractions is much more questionable. Meaningful learning of multiplication, division, decimals, fractions, ratio and proportion builds on the concept of rational number. Essentially what this means is that the nature of the unit has now changed. No longer are all quantities represented in terms of units of 'one' (as they are in natural number). The unit can now mean composite units or, indeed, partitioned units on which the task of enumeration is a useless exercise (for example, one cannot count things to generate a fraction nor can one use counting based algorithms to sequence fractions). If children are to meaningfully engage in the formal work which is typically suggested for the upper primary school (SOED, 1991) they require to reconceptualise what a 'number' can now mean. And yet nowhere in the audit reports was there any explicit recognition that such reconceptualisation is critically important to the continuation of meaningful learning of Numeracy. For example, while there were frequent references to the varying levels of proficiency in the "recall of multiplication tables" or in the "written calculation of decimals, fractions and percentages" there was no reflection of multiplication being anything more than 'repeated addition' or division being anything more than 'sharing'. While these notions of multiplication and division are not necessarily wrong, they are primitive because they depend only on the concept of number being understood as a natural or whole number which constrains numerical thinking (Fischbein *et al*, 1985). The dominance of natural number can leave the child with a range of misconceptions, some of which are that 'you can't divide by a larger number', that '$\frac{1}{2}$ is less than $\frac{1}{4}$ because 2 is less than 4' that 'fractions are not numbers' and that 'the answer to 5.3+2.42 is 2.95' (Owens, 1993). Given such misconceptions it is perhaps not surprising that segments such as "pupils in the upper stages had difficulties with decimal calculation", "some P7 pupils had difficulty adding simple vulgar fractions", "some P6 did not know the meaning of the decimal point" were common while segments such as "some had grasped the relationship between decimals and fractions" and "the most able showed a grasp of decimal place" were much less common. What this seems to point to is that the Inspectorate's conception of Numeracy is not explicitly tied to a mathematical analysis of the developmental nature of Number. If one's conception of Numeracy is not primarily based on the idea that becoming numerate is an intellectual achievement which begins early in life, which operates with representations of objects (as distinct from the objects themselves) and in which the essential object, a number, has a series of increasingly more differentiated meanings, then it is probably quite reasonable for Numeracy to be thought of a the mastery of a system of symbols which are manipulated according to predetermined rules and procedures.

CONCLUSION

In a study designed to infer the 'official' conception of Numeracy (as published in Scottish HMI audit reports), it is clear that computational proficiency is a very significant part of what it means to be Numerate. While the understanding of number is also part of what it means to be Numerate, the emphasis placed on understanding appears to be much less than that placed on proficiency. The implications of this finding are alarming. If children's proficiency in computation is beyond their level of conceptual understanding, many of them are getting correct answers to problems that they do not understand. This disjunction is of concern (as indeed it has been for many years) and would seem to be in urgent need of examination. If understanding is a significant part of what it means to be Numerate, then finding ways of enabling understanding to develop would seem an appropriate matter for educational research to address. Numerical understanding, it has been argued here, is about making connections between different pieces of knowledge: between informal and formal knowledge, between extant and new knowledge and between procedural and conceptual knowledge. One question to which we now need answers is, 'How might these different pieces of knowledge be related?' When we better understand the nature of the relationship, a second question which arises is, 'How can formal pedagogical practices better support the development of Numerical understanding?' These are clearly difficult and complex questions but they are ones to which we need answers if we are to seriously address the concerns that children's Numerical achievements reflect a lack of flexibility in handling number.

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