

Why the journey to mathematical excellence may be long in Scotland's primary schools

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ABSTRACT

In the past 30 years mathematics curriculum reform in many countries has attempted to replace procedural approaches to learning the subject with inquiry based learning that hopes to develop mathematical thinking. Scotland's Curriculum for Excellence can be seen as part of this trend. The success of such moves is highly dependent on the learning experiences devised by teachers which in turn are informed by their own conceptual understanding of the content and processes of the subject. As mathematics curriculum reform has shifted from "an emphasis on knowing things to being able to do things" (Yates & Collins 2010: 89) subject content detail has tended to be removed from curriculum documentation to be replaced by more global outcomes when it is the very detail teachers may need to prompt and augment their own subject knowledge.

This study analysed data from national and international surveys of achievement to gauge how well prepared Scottish children are in mathematics. It also examined subject content and advised pedagogies in previous and current mathematics curriculum documents to ascertain how these have changed over time. On this basis, recommendations are made relating to national qualifications, initial teacher education and continuing professional development. The hope is that these can begin to inform future policy development on how best to prepare teachers to deliver mathematics within Curriculum for Excellence in such a way that will meet the high aspirations Scotland has for its young people.

INTRODUCTION

Recently in this journal Priestley (2010) questioned whether Scotland's *Curriculum for Excellence* (Scottish Executive Education Department (SEED) 2004) would result in transformational change or business as usual and in an earlier paper he stated,

'Teachers must clearly understand reform and have the prerequisite skills to put it in place, if they are to enact it successfully.' (Priestley, 2005: 36)

In this paper the focus is on mathematics; the same question is asked about whether the current reform will result in change and it is suggested that for it to be enacted successfully, addressing the subject knowledge of some primary teachers must be a key consideration.

Stenhouse (1975) noted that curriculum can be viewed as intention and as reality. In the past 30 years mathematics curriculum reform in many countries attempted to replace procedural approaches to learning the subject with inquiry based learning that promoted conceptual understanding, for example, in the UK (Cockcroft 1982), in the USA (National Council of Teachers of Mathematics 1989), in Australia (Australian Education Council 1991), in New Zealand (Ministry of Education 1993) and in China (Ministry of Education 2001). While current curriculum documents in many of these countries have developed this guidance further, and Scotland is no exception with its *Curriculum for Excellence*, the success of these is highly dependent on the learning experiences devised by teachers which in turn are informed by their own conceptual understanding of the content and processes of each subject. In particular, mathematics curriculum reform that has shifted from "an emphasis on knowing things to being able to do things" (Yates & Collins 2010: 89) has tended to remove content detail from teachers who may need this very guidance to prompt and augment their own subject knowledge.

While acknowledging like many before me (Ashcraft 2002; Hembree 1990; Ingleton & O'Regan 1998; Martinez & Martinez 1996) that the cognitive aspect of teaching is intimately related to affective factors, such as confidence in, attitudes to and beliefs about the subject, this paper addresses only the cognitive. This is for two reasons: first, to maintain a focus on what I argue are the key issues; and second, because of literature suggesting that tackling the cognitive in the first instance may in fact lead to more positive attitudes and confidence (see, for example, Grossman 1985; Henderson 2012; Pajares & Miller 1994). While the situation in Scotland provides the focus for this paper, the central message of assuming educational change can be driven by curriculum reform alone is more international. There may be lessons here for many countries considering their own mathematics curriculum.

THEORETICAL CONTEXT

Firstly, I shall describe the theoretical context in which I base my argument, with a brief exploration of the constructivist model of learning and teaching and mathematical knowledge for teaching.

Constructivism

Constructivism proposes that children construct knowledge about the world around them. This knowledge is shaped, developed and refined as they interact with others and their environment. Cognitive constructivism stems largely from the work of Piaget and his followers, Bruner among them, and focuses on the individual construction of knowledge, while social constructivism derives from the work of Vygotsky and holds that the social environment plays a central role in learning (Liu & Matthews 2005). Constructivist teachers see it as their role to create learning contexts which will facilitate this and indeed they are often described as facilitators of learning.

When looking at how one teacher viewed constructivist approaches compared to the transmission model of teaching she had favoured previously, Wood and Cobb (1991) reported that she changed her beliefs about the nature of mathematics from being a set of rules and procedures to meaningful activity. She encouraged her children to be interactive and communicating, when in the past they had been passive, and rather than the authority within the classroom resting with the teacher and textbooks she became the initiator and guide through the students' development of knowledge. Boaler (1997) studied how traditional and constructivist approaches affected learners of mathematics when she compared what she termed a 'traditional' school, Amber Hill, and a 'progressive' school, Phoenix Park. The mathematics teaching in the former emphasised set methods and procedures, kept topics distinct from each other and closed down mathematical problems. The approach at Phoenix Park, on the other hand, was open, gave a degree of choice to its pupils and was concerned about giving them mathematically rich experiences. In a series of assessments where the Amber Hill pupils performed worse than their counterparts in Phoenix Park, Boaler (1997) argued that the two schools had developed different kinds of knowledge in their pupils. Pupils in the first school had a broad understanding of facts, rules and procedures, but they found it difficult to remember these over time, while those in the second school were flexible and adaptable in their mathematics and able to use their knowledge in different situations. It was not the case that the Amber Hill students had learned less mathematics, they learned different mathematics through the practices employed at the school. The Phoenix Park practices appeared to result in more secure knowledge that was transferable across a number of situations. Boaler concluded that attempts to transmit knowledge to students are less helpful than classrooms where children are "apprenticed into a system of knowing, thinking and doing" (1997:109).

Research has been carried out into gains in achievement when constructivist models are used for teaching mathematics. The Cognitively Guided Instruction (CGI) programme (Carpenter *et al.* 1996) featured children working together in small groups while the teacher posed problems and then circulated to scaffold them in their progress. It was found that

increased student achievement on computational skills and higher level thinking occurred. Another research project in the United States concluded that pupils of constructivist teachers developed in an atmosphere of trust that resulted in pupils being enthusiastic and persistent in mathematical problem solving (Cobb 1995). Here in the UK the constructivist approaches used in the Cognitive Acceleration in Mathematics Education (CAME) project resulted in some gains in student achievement (Shayer *et al.* 1999).

Mathematics knowledge for teaching

As has been discussed earlier, the constructivist principles apparent in reform pedagogies have shifted the responsibility of how mathematics is taught to the teacher. Curriculum advice is just that, advice. Gone are details of the knowledge to be imparted and in their place are sometimes vague statements of outcomes, examples of which are given later in this paper, for which teachers have to create learning experiences. Even if we accept that this move is desirable, and it is beyond the scope of this paper to argue the case, it is clear that the transmission model of teaching may no longer result in the type of relational understanding (Skemp 1976; 1989) that is now required.

For the past 30 years or so successive attempts have been made to define the knowledge base for teaching (Shulman 1986; 1987) and, more specific to this discussion, the mathematical knowledge base for teaching (Ball *et al.* 2008; Fennema & Franke 1992; Rowland *et al.* 2005). It is interesting to note that content knowledge came top of Shulman's (1987) categories of teacher knowledge needed to promote understanding in their pupils and who could argue with Derek Haylock's assertion that,

"One of the best ways for a child to learn mathematics is for a teacher who understands it to explain it to them" (2010:3).

Of the seven categories of knowledge outlined by Shulman (1987) three can be thought of as specifically relating to content: content knowledge, or subject matter knowledge (SMK) as Shulman (1986) had previously described it, pedagogical content knowledge (PCK) and curriculum knowledge. The first of these, SMK, has been a fertile ground for researchers who in many cases found it wanting (Askew *et al.* 1997a; Ball & McDiarmid 1990; Hill *et al.* 2008; Ma 1999), with the resulting implications for teacher effectiveness. In two of these studies, Askew *et al.* (1997a) reported that pupils of primary teachers with a low proportion of conceptual links in their mathematics SMK made the lowest gains in attainment, while Ma (1999), on trying to discover the possible causes of unsatisfactory mathematics achievement of children in the USA, concluded that in order to improve mathematics education the quality of teachers' SMK needed to be improved.

Anecdotally we may know teachers who are excellent mathematicians but whose teaching skills leave a lot to be desired. So while sound content knowledge is a necessary condition for effective mathematics teaching, it is not sufficient. It was in recognition of this that Shulman (1987) also identified pedagogical content knowledge (PCK) as being of special interest as it is essentially what a teacher does. It takes the content knowledge a teacher possesses and combines it with professional understanding of how children learn; it is a blending of subject knowledge and pedagogy which transforms topics from mere content to a form which appeals to the different abilities and interests of learners. Effective teachers of mathematics must be able to move back and forth between the mathematics and the pedagogy (Steele 2005), drawing on both to meet the needs of the learner. Illustrative examples of the consequences of incomplete subject knowledge and the impact this has on pedagogical approaches abound in the literature. A fifth grade (age 10) teacher in a study by Heaton (1992) was able to set children the pedagogically sound activity of creating a park with a \$5000 budget so that several aspects of mathematics could be taught in a meaningful context. However, they went on to teach the students that the perimeter of a rectangle is calculated by multiplying the length and breadth; subject knowledge prevented the sound pedagogy from producing a successful outcome for the learners. One of the US teachers in

Ma's (1999) study tried to explain the problem of $1\frac{3}{4}$ divided $\frac{1}{2}$ by asking what $1\frac{3}{4}$ pizzas by $\frac{1}{2}$ pizza would be. One of the Chinese teachers, on the other hand, used the question that half the length of a piece of fabric was $1\frac{3}{4}$ metres, what would the whole length be? The US teacher understood that pedagogically a convincing representation of the problem could lead to sound learning, but lack of understanding of the content meant that the resulting example was nonsense. The Chinese teacher was able to combine sound subject knowledge and that of pedagogy to come up with an example that was capable of developing understanding in the pupils.

The third category of content knowledge, curriculum knowledge, as the name suggests is knowledge of the curriculum and instructional resources that can be used to deliver it. Making decisions about the content to be learned and how to organise that learning in terms of materials, texts and approaches is the "stuff of the teaching profession" (Hawthorne 1992:1) and the way the teacher interacts with the children through a task will shape the type of mathematical thinking that occurs (Osana *et al.* 2006). This interaction with children in specific classroom contexts is reflected in a conceptualisation of mathematical knowledge for teaching proposed by Deborah Ball and her team at the University of Michigan (Ball *et al.* 2008) which breaks down PCK further into knowledge of content and students (KCS) and knowledge of content and teaching (KCT). This model better reflects the dynamic nature of teaching that recognises the interaction between teachers and learners, where teachers' subject knowledge can develop as a result. While the University of Michigan team's current work is attempting to discover whether these constructs do in fact exist as viable models for learning more about mathematical knowledge for teaching, it is clear that knowledge of subject content is key to both.

THE CURRENT SITUATION IN SCOTLAND

Mathematics achievement

In 2004 the Scottish Executive Education Department (SEED) launched *Curriculum for Excellence* (SEED 2004) which seeks to provide 'a coherent, more flexible and enriched curriculum' (Scottish Government 2009a: 3) for children from the ages of 3 to 18. The predecessor of CfE, *5-14*, provided detailed mathematics content and progression for children throughout primary school and the first two years of secondary school. In addition it suggested a range of pedagogical approaches for how the subject should be taught (Scottish Office Education Department (SOED) 1991) based on the constructivist principles already discussed. The effectiveness of these approaches can be judged by studying the two main surveys used in Scotland to assess mathematics performance, nationally the Scottish Survey of Achievement (SSA) and internationally the Trends in Mathematics and Science Survey (TIMSS). The SSA assesses children in various areas of the curriculum at Primary (P) 3 (7-8 years), P5 (9-10 years), P7 (11-12 years) and Secondary (S) 2 (13-14 years). The main findings in the 2008 survey (Scottish Government 2009b) were that at P3 87% of children were estimated to have 'well-established' or 'very good' skills at the expected 5-14 levels in mathematics. However, the proportion of pupils attaining the expected levels in mathematics decreased to 47% by P7 and by S2 that figure had dropped to 30% for the expected level for the stage. It is clear from this that as children progress through the primary school and into the early stages of secondary all is not as it should be in terms of mathematics attainment.

While the SSA in mathematics is conducted biennially TIMSS seeks to assess mathematics and science competence at grades 4 (P5) and 8 (S2) internationally every four years, as well as collecting information about the educational contexts for learning the two subjects. To enable comparisons across TIMSS assessments a scale average of 500 is used, with the scale based on the 1995 assessments and methodology. The international average, on the other hand, has to be re-calculated for each new cycle of assessments. Scotland's average scale score in 2007 was 494 at grade 4 (P5) and 487 at grade 8 (S2), making the latter significantly lower than the TIMSS scale average (IEA 2008), thus

confirming the SSA findings that as children progress through the primary school and into the lower stages of secondary, attainment in mathematics falls. Eleven of the 16 OECD countries scored higher than Scotland. In addition, the gap between the lowest and highest performers in P5, measured using the inter-quartile range, has narrowed since 1995 due to an increase in the score of low performers but also a decrease in the score of high performers (Scottish Government 2008). When the TIMSS Advanced International Benchmark of 625 is considered (IEA 2008), our top performing P5 pupils achieved significantly lower than they did in 1995.

These measures of performance paint a mixed picture of Scotland's ability to hold its own in comparison to other nations when average mathematical ability is measured, and cast doubt on our achievements at the higher levels. Evidence presented would seem to indicate that some change in how mathematics is taught would appear to be justified. Before consideration of how this change can be brought about, it is necessary to look at the current situation regarding student primary teachers' mathematics knowledge. Studies at the author's institution have reported that student teachers' mathematics subject knowledge was often lacking when assessed using an online assessment and that confidence levels could be low (Henderson & Rodrigues 2008; Henderson 2012). If the subject knowledge of these future teachers is lacking, how likely is it that they will be able to develop into autonomous professionals who adopt constructivist models of teaching mathematics by creating motivating learning experiences through which children's mathematical understanding will develop?

RADICAL REFORM OR BUSINESS AS USUAL?

Curriculum for Excellence has been lauded as 'the most radical reform of education in Scotland for a generation (Scottish Government 2009c: no page) yet in terms of mathematics how different is it? One change is that a set of numeracy outcomes exists that is distinct from those in mathematics. Numeracy, along with literacy and health and wellbeing, are the responsibility of all teachers, but as primary teachers are responsible for all aspects of the curriculum anyway this does not constitute real change at this stage. The 5-14 mathematics curriculum guidelines (SEED 1999; SOED 1991) included a clear progression of knowledge arranged across six levels, A-F. For example, under the Number, Money Measurement outcome and the strand of Round Numbers the progression is shown in table 1.

Table 1: 5-14 progression of rounding numbers

Level B	Level C	Level D	Level E	Level F
Round 2-digit whole numbers to the nearest 10.	Round 3 digit whole numbers to the nearest 10 (e.g. when estimating)	Round any number to the nearest appropriate whole number, ten or hundred	Round any number to one decimal place	Round to a required number of decimal places and to a required number of significant figures

Source: adapted from SOED (1991); SEED (1999)

The knowledge that pupils should have at each level is clearly stated and a framework for conceptual development is provided.

To remind readers, *Curriculum for Excellence* is arranged over five levels, Early to Fourth. Early Level is two years of nursery and P1, First level is P2-P4, Second Level is P5-P7, Third Level is S1 and Fourth Level is S2-S3. There is currently indecision about whether S4-S6, will be included as a Fifth Level, despite aims of a seamless 3-18 curriculum. First, Second and Third Levels would broadly compare with 5-14 Levels B-F. The equivalent outcomes in the new curriculum are shown in table 2.

Table 2: *CfE* progression of rounding numbers

First Level	Second Level	Third Level
I can share ideas with others to develop ways of estimating the answer to a calculation or problem, work out the actual answer, then check my solution by comparing it with the estimate.	I can use my knowledge of rounding to routinely estimate the answer to a problem then, after calculating, decide if my answer is reasonable, sharing my solution with others.	I can round a number using an appropriate degree of accuracy, having taken into account the context of the problem.

Source: adapted from Scottish Government (2009d)

What should be apparent from tables 1 and 2 is that the clear detail of knowledge to be taught and learned that 5-14 offered has been replaced with vagueness. A teacher whose own understanding of the topic is less than secure may be challenged by what is meant by 'I can use my knowledge of rounding' from the Second Level in table 2. What knowledge of rounding is required at the Second Level and how has it developed and progressed from the First Level? At the Third Level, what is an appropriate degree of accuracy? Is it one decimal place, one significant figure, the nearest whole number? As the clear progression that existed under the 5-14 curriculum will no longer be provided it is up to the teacher to fill in the missing detail and this requires a degree of understanding about and confidence in the topic that may not be present.

While the learning outcomes of *Curriculum for Excellence* are quite different from the attainment targets of its predecessor, many of the constructivist teaching approaches recommended were already present in 5-14 (Henderson & Cunningham 2011) and, in fact, advice to move to such approaches had been recommended even earlier by Cockcroft (1982). However, most mathematics lessons in Scotland still tend to feature some form of teacher-led demonstration followed by children individually practising skills and procedures from a commercially produced scheme (SEED 2005); compared to the international average of 65% and 60% for P5 and S2 pupils, 72% of both were taught using a textbook as the primary resource (IEA, 2008) and pupils using textbooks and working quietly on their own is the most common form of activity in mathematics classes in Scotland (Scottish Government 2009b). This would seem to suggest that attempts over more than 25 years to move to constructivist models of teaching mathematics have not been entirely successful. Add to this the fact that content detail has now been removed from curriculum guidelines and it becomes clear that little may change in how mathematics is taught under the new curriculum.

DISCUSSION AND RECOMMENDATIONS

In this paper it has been argued that curriculum reform which attempts to increase teacher autonomy in how a subject is taught without also considering that teacher's own subject knowledge may have little chance of success. Although much of this discussion has centred round the current curriculum changes in Scotland, there are wider implications. Changes to the pedagogy of mathematics were central to the predecessor of *Curriculum for*

Excellence, 5-14, yet the teaching of mathematics continued in much the same way as it had prior to its adoption with the transmission model to the fore. In order to ensure that curriculum reform in mathematics translates into practice a three-pronged approach is suggested with implications for national qualifications, initial teacher education and continuing professional development (CPD).

National qualifications

When studies confirm the importance of subject knowledge, the focus of policymakers often turns to the appropriateness of entry qualifications to teacher education courses.¹ If teachers' subject knowledge is not sufficient it would seem natural to call for an increase in the level of these qualifications. However, Henderson and Rodrigues (2008) and Henderson (2012) reported that student primary teachers with the more advanced Scottish Higher (SCQF level 6) were even less likely to display competence in primary mathematics in an online assessment of knowledge and skills than their peers with the lower Standard Grade Credit or Intermediate 2 qualification. This suggests that raising the entrance qualification may not be the answer. It appears that this is not just a Scottish phenomenon, as studies have been reported previously in both the USA and England. Thirty years ago Begle (1979) reported that US teachers who had taken advanced mathematics courses (calculus and beyond) had positive effects on pupils' achievement in only 10% of cases, and more worryingly negative effects in 8%. Monk's (1994: 130) work confirmed much of Begle's (1979) findings and reported that undergraduate courses in mathematics pedagogy 'contribute more to pupil performance gains than courses in undergraduate mathematics'. In England Askew *et al.* (1997a) found that pupils of teachers with advanced level mathematics were less likely to make positive gains in attainment than their counterparts with the lower level required for entrance to teacher education programmes.

It may seem like a contradiction to the central argument that more advanced mathematics qualifications may not be the answer to improving subject knowledge. However, a more dedicated qualification (Advanced Subsidiary (AS) level equivalent to SCQF Level 6), such as that proposed by Burghes (2009), could provide a way forward. This would omit many aspects of the subject currently present in the Scottish Higher – for example, calculus and logarithms. It is unlikely that their study adds to the connected picture of mathematics seen as necessary for it to be taught effectively (Askew *et al.* 1997a) as they are simply too advanced in nature to be relevant in the primary curriculum. The time freed by their omission would allow for in-depth inquiry into such primary level topics as the Fibonacci sequence and the Golden Section, the different calendars used in the world, number systems and patterns and the use of primes. It is study of topics such as these that might lead to an 'at-homeness' with mathematics, to borrow a phrase first used by Cockcroft (1982: 11), or 'a mathematical sensibility' as described by Askew (2008: 22). If, as reported by Macnab and Payne (2003), student primary teachers are more apprehensive about teaching mathematics than any other curricular area, such a qualification may deepen understanding and lead to increased subject confidence. While a case could be made for requiring that entrants to primary education programmes have this proposed qualification, another option would be to ensure the content is covered as part of initial teacher education.

Initial teacher education

In Scotland there is no consistent approach to addressing mathematics subject knowledge during initial teacher education. In England teacher education institutions audit students' mathematics knowledge and address any deficiencies. Upon completion of

¹ Students in Scotland enter primary education programmes with minimum mathematics qualifications at Scottish Credit and Qualifications Framework (SCQF) level 5 (SCQF 2003) - see Appendix 1 for a breakdown of school-based national qualifications in Scotland.

education courses newly qualified teachers then have to pass an online numeracy assessment before qualified teacher status is achieved.

The recent review of teacher education in Scotland (Donaldson 2011) recommended that applicants to primary education programmes be assessed in literacy and numeracy at the interview stage, that any deficiencies in their knowledge be addressed during their programmes and that they demonstrate a high level of competence by the end of these. Already at the author's institution all student primary teachers are required to reach and maintain an 85% competence threshold in an online mathematics assessment. It is hoped that this tool, which offers formative feedback when a question is answered incorrectly, in association with workshops on the pedagogy of mathematics, and the students' experiences while on school placements, will contribute to a transformation of their subject knowledge into effective PCK. A study across all four years of the undergraduate primary education programme revealed that students who engaged with this process achieved increasingly higher scores and their levels of confidence in mathematics improved (Henderson 2012). At the risk of oversimplifying what is a complex process, this upward spiral of competence and confidence is surely what teacher education must strive to achieve.

Of course, maintaining sound subject knowledge and developing deeper understanding of the subject can take many more years than those spent in initial teacher education and so consideration must also be given to the continuing professional development of teachers.

Continuing professional development

Research into mathematics continuing professional development (CPD) has produced interesting results. Askew *et al.* (1997b) reported that highly effective teachers were more likely to have been involved in extended mathematics CPD, with sessions of 10-20 days likely to have more impact. The researchers concluded that non subject specific CPD was not the answer. While there was recognition that there may be transferable skills which cut across subject areas, the recommendation was that mathematics-focused CPD is the best way to accomplish sustained change in practice that impacts on children's attainment. This was confirmed by Kennedy (1998) who found that subject-specific CPD was more effective in raising attainment than generic professional development.

One way of ensuring that school-based CPD could be run by subject specialists would be the introduction in Scotland of a form of the Mathematics Specialist Teacher (MaST) programme currently offered at some English universities. These Masters-level courses were created as a result of an independent report into mathematics teaching in primary schools in England (Williams 2008) which recommended that within the next 10 years each primary school should have at least one mathematics specialist.

In Scotland, current Masters levels programmes for teachers, most commonly the Master of Education (M.Ed.), offer mostly generic courses. A Masters level programme entitled Developing Mathematical Thinking in the Primary Classroom is currently being piloted at the author's institution and it is the first mathematics module at this level available in the country. Until now the only way for a Scottish teacher to develop their interest in mathematics is to undertake private study or to engage in local authority or national events, where available. Given the constraints on public finance it is likely that such opportunities may be few in the future and so developing 'maths champions' to lead local mathematics CPD may be the way forward in Scotland also.

CONCLUSIONS

In this paper I have attempted to argue that curriculum reform in Scotland may not lead to the hoped-for change in practice in mathematics unless teachers' subject knowledge is also addressed through national qualifications, initial teacher education and continuing professional development. Few people would argue that the aims of *Curriculum for Excellence* are not admirable in that they set out an ambitious vision for education in twenty-first century Scotland. However, previous curriculum reform in mathematics in the shape of

5-14, which had already advocated many of these reforms, did not lead to change and, without addressing primary teachers' own subject knowledge, it has to be questioned whether *CfE* will either. One of the aims of the new curriculum is that Scotland's teachers will have more autonomy regarding what, when and how they teach. Thorough knowledge and understanding of curricular content is more likely to lead to this, than a curriculum that blurs that content and may leave teachers feeling less confident about what they are to teach. Handal and Herrington (2003) argue that it is teachers who ultimately decide the fate of any curriculum reform. This has been seen in Scotland with the minimal move to adopt reform practices in the teaching of mathematics in the past 20 years. If teachers are to be regarded as self-determining professionals their confidence to implement the new curriculum has to be supported and one way to do that may be to ensure their subject knowledge is deep, vast and thorough (Ma, 1999). Without this the road to mathematical excellence in Scotland's primary schools may indeed be long.

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APPENDIX 1

SCQF Levels	SQA Qualifications
7	Advanced Higher
6	Higher
5	Intermediate 2/ Credit Standard Grade
4	Intermediate 1/ General Standard Grade
3	Access 3/ Foundation Standard Grade
2	Access 2
1	Access 1

Source: adapted from SCQF (2003)

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