MATHEMATICS EDUCATION IN SCOTTISH SCHOOLS: AN UNCERTAIN VISION?

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SYNOPSIS

The SOEID Assessment of Achievement Programme (AAP) surveys of pupil attainment in mathematics in Scottish schools at primary 4, primary 7, and secondary 2 (roughly ages 9, 12, and 14) have revealed a decline in performance over the years in which testing has been carried out. The mathematics results of the 1994/5 Third International Mathematics and Science Study (TIMSS) at broadly ages 8/9 and 13/14 placed Scotland below midway for both age ranges, in the performance table of the countries tested. Within the last decade a number of Scottish Office Reports have been published dealing with both curriculum and methodology in mathematics teaching. This paper examines values and purpose in mathematics education in Scottish schools in the light of AAP and TIMSS data and the rhetoric of policy documents. The paper forms part of a wider investigation into the processes of change in education, with a particular focus on mathematics.

INTRODUCTION

In 1996/7 the Scottish Office Education and Industry Department (SOEID) published and disseminated widely two summaries of the results of the 1994/5 Third International Mathematics and Science Study (TIMSS), as they related to Scotland (SOEID, 1996a, 1997a). TIMSS is the most extensive international study of mathematics and science curricula and attainment ever conducted, building on a previous study in the early 1980s. Surveys of the comparative results for the participating countries are contained in Martin *et al* (1996, 1997). Overviews of earlier international surveys of attainment are contained in Brown (1996) and Reynolds (1996). The latter, while written from an English point of view (as an OFSTED publication), in its international aspects has equal relevance for Scotland. An account of Scottish performance in the 1991 international survey of mathematics attainment is contained in SOED (1992).

TIMSS participating countries were asked to select pupils from the two years containing most 9 year olds and the two years containing most 13 year olds. For Scotland these were Primary 4/5 and Secondary 1/2. Scotland was placed 15th out of 24 countries for Primary 4 pupils and 16th out of 26 countries for Primary 5. At secondary level, Scotland ranked 26th out of 39 countries for Secondary 1 and 28th out of 41 countries for Secondary 2. Scotland's relatively poor international standing was highlighted both by national newspapers and the educational press, thus bringing the matter to the attention of the general public.

At roughly the same time as the TIMSS Study, the 1994 SOEID Assessment of Achievement Programme (AAP) survey of mathematical attainment in Scottish schools at stages primary 4, primary 7, and secondary 2 (Robertson *et al*, 1996) revealed a continuing decline in performance, particularly evident in the topics of number, money and measurement, compared with the 1991 survey which in turn documented a decline from 1988 (Macnab *et al*, 1989; Robertson *et al*, 1992). The most recent (1997) survey (Howat, R., Smith, D., 1998) reveals a more complex picture. The results of these internal surveys were rather less well reported in the press, with the exception of the 1988 survey which was the first to identify the downward trend, in this case from 1983. One of the reasons for the greater publicity

given to the 1988 results was the apparent lack of any predictor evidence so that the results came as an unwelcome surprise to many of those who took a favourable view of the state of Scottish mathematical education at that time. Perhaps not surprisingly, attempts were made to explain away the results on a variety of grounds, for example, curriculum drift, invalid comparisons, and inappropriate statistical analysis. By the time (1996) of the publication of the 1994 AAP survey results, the educational climate had changed and the Report attracted little public comment. The public view of HM Inspectorate of Schools was encapsulated in the following three statements (Clarke and Moffat, 1996, p 5).

> Mathematics education (in Scotland) is basically sound.
>
> There is cause for concern about specific aspects.
>
> We must not be complacent but should aim to improve performance.

In the present decade, there have been three major Scottish Office documents dealing with the teaching and learning of mathematics in schools (SOED, 1991; SOED, 1993; SOEID, 1997b). The first of these, *Curriculum and Assessment in Scotland: National Guidelines Mathematics 5–14* (CASM) (SOED, 1991) lies within the Scottish equivalent of the National Curriculum in England and Wales, although it is an advisory document without the force of law, as distinct from the position south of the border. The Guidelines were preceded by the issue of a consultation document, Working Paper 3–Mathematics (SED, 1990), produced by a Review and Development Group (RDG) set up under the auspices of the Scottish Consultative Committee on the Curriculum (SCCC). The RDG carried out a "wide-ranging review of good practice in order to set out clearly the knowledge, understanding, skills, and attitudes appropriate to its curricular area", and advised on "the formulation of national curriculum guidelines which would identify the aims of study, the ground to be covered, the way learning should progress, and how pupil's attainment should be monitored and recorded" (SOED, 1991, p.vi). No reference is made in CASM to the results of the 1988 AAP survey.

The second document *Effective Learning and Teaching in Scottish Secondary Schools–Mathematics* (ELTM) (SOED, 1993), derives (as stated in its foreword) "from the inspection of more than 200 mathematics departments conducted since 1983, and from the monitoring and evaluation of developments in the teaching of mathematics". ELTM contains an extended discussion of all aspects of the organisation and management the teaching and learning of mathematics in secondary schools, including an initial section on aims and purposes. ELTM also contains an extensive list of recommendations about which the summary introduction says, "All those concerned with the process of mathematical education will wish to review their practices in the light of the recommendations made." However, in general, ELTM portrays mathematical education in Scottish schools as broadly satisfactory.

The third document, *Improving Mathematics Education 5–14* (IME) (SOEID, 1997b), followed on from the results of the TIMSS survey and the 1994 AAP survey, both of which reported in 1996. In distinction to ELTM, the IME report deals with both Primary and the early years of Secondary schools. IME also relates to two reports dealing more generally with standards in Scottish schools, *Standards and Quality in Scottish Schools 1992–95* (SOEID, 1996b), and *Achievement for All* (SOEID, 1996c). While many of the recommendations in IME mirror those in ELTM, there is a direct emphasis on four issues:

> moving from mixed-ability teaching to some form of setting by ability;
>
> moving from individualised approaches to learning to more teacher-led whole class activity;

reducing dependence on the calculator;

increasing pupils' facility in mental arithmetic.

In contrast to the relatively confident tone of ELTM, the IME report states that,

> comparison of performance and practice in Scotland with that of other countries underpins this report on improving mathematics education 5–14 and the Agenda for Action which it offers. Implementation of the Agenda should lead in time to an improvement in the performance of mathematics of pupils in primary schools and in S1/2 (Para 1.8).

A further report, *Achieving Success in S1/2*, was published later in 1997 than IME and "takes forward specific advice about mathematics contained in the IME report" (SOEID, 1997c, p.4).

Thus, in the period from the late-eighties, there has been no shortage of evidence on standards of mathematical performance of pupils in Scottish schools, or of advice and guidance from Scottish official sources on the learning and teaching of mathematics.

PURPOSE IN THE MATHEMATICS CURRICULUM IN SCOTTISH SCHOOLS

In examining purpose, values, and vision, in mathematics education it is useful to follow the structure adopted by TIMSS and others and identify three conceptual aspects or levels in the process of curriculum development and implementation. The first of these, *the intended curriculum*, refers to the curriculum as described in official documents, for example, the National 5-14 Guidelines for Mathematics. The second level, *the implemented curriculum*, concerns the implementation of official documentation in the context of individual schools and classrooms. The implemented curriculum is influenced by a variety of factors including internal school organisation, views on topic sequencing, and the ideas, attitudes, and beliefs of teachers. The third level, *the attained curriculum*, describes the attained outcomes of the educational process - i.e., what pupils have learned. The attained curriculum deals with knowledge, skills, and understanding. More detailed descriptions can be found in Robitaille *et al* (1993, 1996), the latter in particular.

It is, however, of value to add a fourth curricular level to these three, *the experienced curriculum*. This term refers to pupils' classroom experiences of mathematics as they proceed through their school years. It relates to how they feel about their mathematical experiences, what meaning they attach to them, what value they find in them. Clearly the experienced curriculum owes much to the implemented curriculum and in turn has a major influence on the attained curriculum. It also owes a great deal to the class teacher's empathy both with the subject and with the pupil as learner. These four aspects of the curriculum are not new; in the TIMSS context the first three derive from the first IEA International Study in 1976 (Travers, K. J. *et al* 1989), which in its turn based its thinking on ideas developed in the 1960s. More generally, the concepts embodied by all four may be found for example in Hass (1980) which also dates back to the 1960s.

The three most recent SOEID reports, *Achievement for All*, *Improving Mathematics Education 5–14*, and *Achieving Success in S1/2*, indicate clearly that HM School Inspectors in Scotland view current levels of mathematical attainment in Scottish schools up to age 14 as less than satisfactory and in need of serious improvement.

Why should these levels of attainment, as measured by objective surveys (AAP and TIMSS), be deteriorating? One possibility is a mismatch between the structure of the surveys, and school mathematics curricula as actually experienced by pupils.

The construction of both the AAP and the TIMSS surveys reflected the nature of the Scottish mathematical curriculum as officially described, through making use of the guidance of expert advisory groups. Scrutiny of the released item sets confirms this and suggests that any such mismatch must be located at classroom level where the curriculum as experienced by pupils differs from the intended curriculum. A detailed description of the construction of the TIMSS Achievement Tests is contained in Martin and Kelly (1996). A second possible explanation lies in the expectations of the intended curriculum; its structure and goals may be inadequate to produce the desired outcomes of pupil attainment. Yet a third possibility is that the curriculum in its methodological or organisational implementation may not be able to achieve all goals of the vision. In all these possibilities, the intended curricular vision will fail to achieve a structured and effective experienced curriculum. What then is the vision of Scottish mathematical education contained in official reports? What aims and purposes flow from it? Is it consistent over time? We look first at the primary school mathematics curriculum.

THE PRIMARY CURRICULUM

At Primary level the report *Primary Education in Scotland* (SED, 1965), often referred to as the Primary Memorandum, influenced teachers and teacher training courses for a generation; its influence can still be perceived today. An insightful account of how the Memorandum came into being, based on first-hand conversations with its authors, may be found in Farqharson (1985).

In the Memorandum there was a clearly expressed vision of the purposes of mathematics teaching. It was an experiential vision. We read that:

> (pupils) must be given the opportunity to experience the joy and excitement of exploration and the thrill of discovery. Problems should, as far as possible, arise from real situations and lead to activity… they should stimulate discussion and on occasion may require a variety of approaches before a final solution is reached… pupils will progress at times by following their individual interests without regard for a preconceived logical sequence of development… the teacher (must be) frequently prepared to adopt the role of helper rather than of leader and instructor (pp146/7).

The key concepts are "investigating" and "finding out". It is a process-oriented approach, reflected by the positioning of mathematics within the chapter on Environmental Studies.

Now while such a vision is clear and consistent, there are obvious dangers in accepting it at face-value. Nor, at the time of its publication, was it fully intended to be. The report was produced to influence a move away from what was at the time considered to be a mathematics curriculum excessively dominated by extensive arithmetic calculation, learned and performed, to a large extent, by rote methods without full understanding of the reasoning behind the processes involved. Therefore, it gave central prominence to advocating a change of direction. For example, the report comments that:

> for many years the teaching of mathematics in the primary school has been almost exclusively concerned with the development of skill in reckoning and the perfecting of routine methods of carrying out computations. Much of this work has little general educational value (p.145).

However, there was clearly a danger, all too often realised, that the message which reached the classroom was that what really mattered was the mathematical experience, the journey rather than the goal.

While the vision is an attractive one, giving rise to mental images of happy colourful classrooms in which pupils, individually or in groups, are engaged in learning activities geared to their interests and learning readiness, in its practical implementation it suffered from over-optimism about the processes through which mathematical ability is acquired, from a lack of structure and balance between investigation and consolidation, and from the limited mathematical background of most primary school teachers. The resulting experienced curriculum, as a consequence, tended all too often to be an uneasy mix of the old and the new, of investigative procedures and mathematical interest topics not well understood by teachers or pupils as to purpose, overlaid on the traditional algorithmic procedures of arithmetic. The vision, while attractive, was flawed. Until recently, nonetheless, it has been for many an enduring vision of what learning mathematics ought to be like, with many of its recommendations on the broadening of the content of the primary school mathematics curriculum gaining general acceptance.

Eight years after the 1965 report, the Scottish Education Department published *Curriculum Paper 13–Primary Education in Scotland: Mathematics* (CP 13) (SED, 1973). This report, to quote from its foreword, was produced because in the view of HM Inspectorate of Schools,

> there is now a need for a clearer definition of the appropriate content in mathematics for pupils of different ages and abilities.

Curriculum Paper 13 was an attempt to give a structure to the 1965 report by giving practical examples of an investigative approach in action. However, while it provided much welcomed guidance to teachers regarding implementation of that report and indicated clearly the need for careful planning in the structure of the mathematics curriculum, it also reinforced the reduced emphasis on traditional arithmetic. This reduction in emphasis is clearly expressed in paragraph 6 of the introduction, where it is stated that:

> traditionally, arithmetic… has been given pride of place… occupying at least one hour per day and frequently twice as much… Prior to the publication of the primary memorandum (i.e. the 1965 report) the time suggested for mathematics was three hours per week in the Infant Department and four hours per week elsewhere. There is no justification for spending, on average, more time than this on mathematics today and there may be justification for spending less.

Even after allowing for the introduction of decimal currency and the much reduced emphasis on imperial measures in favour of the metric system, such advice, in the context of a much broadened mathematics curriculum, makes clear the diminution in the prominence given to arithmetic. By giving greater classroom practicality to the 1965 vision, CP 13 in essential respects reinforced that vision, setting it into a curricular guidance framework for primary school mathematics in terms of both content and process which prevailed for nearly 20 years.

Seven years later a further report, *Learning and Teaching in Primary 4 and Primary 7* (SED, 1980), was published, based on inspections in a random sample of 6% of all Scottish Primary schools. The report concluded that "the teaching of mathematics has still to deal adequately with branches other than arithmetic and provide pupils with more experience of practical work", and asked "at what expense is a high rate of success in computation being gained?" (p.18). Thus, 15 years after the publication of Primary Education in Scotland and 7 years after the appearance of Curriculum Paper 13, HM Inspectors of Schools found that, in a majority of schools surveyed, the mathematical curriculum "was entirely computation or provided for only rare excursions into the broader fields of mathematics" (p 16). It is clear from

the overall tone of the Learning and Teaching report that, while HM Inspectorate of Schools concluded that "teachers can take satisfaction from the standard that their efforts have achieved" (p 20), they did not regard the mathematical curriculum in most of schools surveyed as satisfactory, with in their view, a continuing overemphasis on computational arithmetic. It is also clear from the report that, from an HMI perspective, the curriculum as implemented by teachers in most schools at that time was not the curriculum intended. This continuing tension between the intended and the implemented curriculum led to increasing uncertainties in many teachers as to goals in mathematics education, resulting in the 1980s all too often in experienced curricula which achieved neither the skills goals of the traditional arithmetic curriculum nor the process oriented goals of 1965 vision.

Why was there this continuing disparity between intended and implemented curricula? Part of the answer would seem to lie in the following view expressed by SED (1980) in the Learning and Teaching Report:

> a considerable number of teachers continued to hold the view that (in mathematics) their job was to concentrate on computational skills, and they had some support from the public, including secondary teachers and parents. Mathematics was regarded basically as a matter of calculating on paper or giving oral answers to questions on mental arithmetic, on tables and the like. (p.16)

However, this cannot be the whole story. To many who were involved in staff development over this period it was apparent that many of those teachers who continued to concentrate on computational arithmetic did so at least partly because it was a curriculum which they felt they could successfully implement and one in which to them the curricular goals were clear in a way in which the goals of the intended curriculum were not. Thus the disparity arose as a result of a combination of an insufficiently clearly articulated intended curriculum and the belief system of teachers. This disparity was the greater where teachers were strongly attainment oriented, that is where a teacher placed considerable importance on the attainment of cognitive learning outcomes, and in particular the attainment of specific mathematical knowledge and skills.

Three years later, the Consultative Committee on the Curriculum (an advisory body reporting to the Secretary of State for Scotland) published a position paper *Primary Education in the Eighties*, produced by the Committee on Primary Education (COPE, CCC, 1983). The focus of the paper is described in the Foreword.

> The paper's starting-point is the Primary Memorandum of 1965; it takes account of subsequent developments, of subsequent reports by HM Inspectors and of a series of papers prepared on aspects of primary education under the auspices of the CCC over a number of years. The paper gathers together these threads, reviews the position reached on the development of primary education in 1983, identifies the main issues and points the way forward.

The COPE report is well-disposed to the Memorandum stating that:

> the Memorandum sets out simply and clearly a philosophy of primary education with which few would argue. It fully deserves its reputation as a watershed document (p 10),

In particular reference to mathematics, COPE felt "free to leave the initiative to others who were doing it well" (p 28). There followed six suggestions, including reducing the dominance of "doing sums", reducing the time allocated to mathematics to around four hours per week, and the development or reinforcement of mathematical concepts and skills within other areas of the curriculum. The overall impression given is that

COPE in 1983 thought that primary school mathematics was in a satisfactory state of development, requiring attention only to some matters of curricular emphasis.

The development of the SOED Curriculum and Assessment programme in the late eighties signalled a fresh attempt at defining and structuring the primary mathematics curriculum. The general aims of the programme were set out in the consultation paper *Curriculum and Assessment in Scotland A Policy for the 90s* (SED, 1987) and gave rise to an extensive debate on structure, purpose and professionalism in Scottish education to age 14 (see, for example, Roger and Hartley, 1990).

Unlike CP 13 in 1973 in relation to the 1965 report, there was no attempt to set the new National Guidelines for Mathematics 5-14 in an historical or developmental context. All that is said is that:

> the advice in these guidelines is based on existing good practice in the design and planning of policies and programmes for the learning and teaching of mathematics in Scottish schools (Introduction p 1).

which raises the obvious question of the criteria used to identify the good practice. The introduction continues:

> this process (of adjusting and developing programmes of work along the advice of the Guidelines) should help to ensure that all pupils experience a coherent, continuous and challenging programme of work, regardless of age, aptitude or physical or social circumstances,

which suggests that, without such adjustment and development, existing programmes of work in many schools were not thought to be achieving that clearly desirable goal.

In their explicit treatment of *Problem-solving and Enquiry* as one of the four strands of the mathematics curriculum alongside *Information Handling, Number, Money and Measurement*, and *Shape, Position and Movement*, the Guidelines left unclear the relationship of the investigational or problem-solving approach to the other three elements of the curriculum, and, by not defining attainment targets for that element, potentially continued the uncertainty regarding the goals of the investigational approach.

While in general the 1991 Guidelines continued to reflect the experiential and individualistic philosophy of the 1965 report, they also signalled clearly that society had a right to expect certain standards of mathematical performance, and that it was the responsibility of the teacher to ensure that these standards were attained. The emphasis thus moved from the individuality of the pupil to general learning expectations within age-groups, resulting in a certain duality of vision between learning as a carefully structured activity designed to lead to predetermined ends, and learning as an open activity with ends determined by the individual pupil.

THE EARLY SECONDARY CURRICULUM

In the early Secondary school somewhat different problems have arisen. Since the introduction of Standard Grade qualification courses in the 1980s following on the "*Munn*" *Report* (SED, 1977), courses of study for pupils in the first two secondary years have lain uneasily between the primary school curriculum and the start of qualification courses in secondary year 3. This is acknowledged, for example, in Paragraph 1.7 of the HMI Report *Learning and Teaching in the First Two Years of the Scottish Secondary School* (SED, 1986).

Before the widespread introduction of mixed-ability class organisation in the early secondary in the late 1960s and early 1970s, secondary school mathematics courses had been differentiated into those suitable for pupils of lower ability and those for pupils proceeding to external examinations in Years 4 and 5. An SED report (SED,

1959), for example, lists no fewer than five different syllabuses for Years 1 to 3, including two specifically designed for girls. The only aims set out for the courses were mathematical in nature, clearly laid out in a straightforward fashion.

The introduction of mixed-ability teaching in S1/2 led to many uncertainties regarding both curriculum and methodology. Overall, it is fair to say that the mixed-ability form of class organisation was introduced for reasons more connected with the phasing out of three year secondary schools and equality of opportunity for all pupils, than as part of a coherent vision for teaching and learning mathematics. (See, for example, MacPherson and Raffe, 1988, Part IV, for a detailed analysis.) Consequentially, much time was spent both in attempting to reconstruct a vision for mathematics education and in its effective implementation.

Twenty years later, in a report on Mathematics in S1 and S2, published by the Scottish Curriculum Development Service (CCC, 1981), there appear both a set of General Educational Aims and a set of Mathematical Aims. Taken as a whole the general educational aims are personal and social in nature while the mathematical aims deal to a large extent with organisational and management matters. Neither individually nor collectively do the two sets of aims set out a clear vision of mathematics in S1/2, as a distinctive educational phase between the primary school and the year 3 qualification courses.

The 5-14 Curriculum and Assessment Development Programme of the late 1980s and early 1990s attempted to rectify this for the early secondary curriculum by treating the years 5 to 14 as a single entity, in effect by grafting S1/2 onto the top-end of the primary school curriculum. S1/2 courses were to be seen as the final two years of a programme of work which commenced in primary 1. In practice this has not yet generally achieved the desired outcome of a continuous and coherent curriculum from primary 1 to secondary 2, due both to the problems involved in the transfer from primary to secondary school, and to the limited nature of the attainment targets set at the upper end of the curriculum. For an account of some of the problems involved, see Simpson (1997) and Simpson and Goulder (1997, 1998).

These problems have had the consequence that in the years following 1991 there continued to be uneven curricular progression from P7 to S1, and that the S1/2 curriculum has continued to lack a clear sense of direction. An attempt has been made to remedy the second problem with the introduction of a new attainment level, level F, beyond the five levels A to E of the original 1991 Guidelines (SOEID, 1999). However, Sections 1 and 2 of the IME Report (SOEID, 1997b) make clear the need to redefine the S1/2 curriculum and provide a vision which provides a worthwhile set of targets for both teachers and pupils.

SCOTTISH MATHEMATICS EDUCATION - A NEW VISION

Prior to the publication of the results of the 1988 AAP survey and to some extent up until the publication of the TIMSS results in 1996/7, mathematics education in Scottish primary schools and the early stages of secondary schools was not giving rise to serious cause for concern within the education profession.

Much of the explanation for this lies in the 1965 Primary Memorandum and the vision it contained, emphasising process over product, and placing mathematics within the framework of environmental studies. The associated educational culture promoted a view of mathematical education characterised by:

- a reduction in emphasis in the development of and practice in formal computation;
- the development of mathematical understanding and skill within a contextualised inductive framework;

- an emphasis on pupils working at their own pace;
- an assessment framework to a large extent dependent on the day-to day observations of the class teacher.

The reduction in formal computation was also a result of the increased availability and use of electronic calculators.

Since these features were regarded as desirable aspects of mathematical education at that time, with an emphasis on the personal development and social aspects of education, it is not surprising that questions of objective attainment were not raised to any marked extent within the profession. There was a view that the investigative, inductive approach would in itself produce mathematical understanding and skill, paralleling the use of similar empirical procedures in other scientific aspects of environmental studies. There was also a body of opinion which took the view that such approaches would make the study of mathematics more palatable to pupils and thus increase their readiness to learn and their liking for the work, which in turn would lead to improved levels of attainment.

Some indication of the conviction with which this outlook was held can be gained by reading, for example, the reactions to the National Curriculum proposals contained in the previously cited Roger and Hartley (1990), and also in Kirk and Glaister (1994), where there is a general emphasis on non-prescription, the individuality of learning experiences, the holistic nature of the educational experience (from which developed organisational and methodological concepts such as the integrated day and thematic studies), and pupil control of learning. Further evidence of the culture conflict engendered by the overall Curriculum and Assessment programme can be found in MacAllister (1993). The general acceptance throughout the 1970s and 1980s of the incomplete vision of the 1965 report, with its emphasis on process and individuality, led to the duality of vision of the 1991 Mathematics Guidelines.

The results of the AAP and TIMSS surveys have forced a further rethinking of the vision towards structure and authority. This period of uncertainty of vision, of what are the fundamental purposes of mathematical education and what is an appropriate pedagogy to realise them, is not yet over, but at least the questions are being asked in a way in which they have not been for over 20 years.

One of the difficulties in capturing a realistic and enduring sense of vision in the teaching and learning of mathematics lies in defining the nature of that vision, and in achieving an appropriate balance between its internal and external aspects. From the mid-sixties onwards until the IME Report, the dominant climate of opinion has sought to place mathematics within a broader curricular context, starting with its inclusion in the 1965 Primary Education report as part of Environmental Studies and reinforced by documents such as CCC (1981) and CCC (1983). A vision for mathematics teaching and learning which does not have at its heart an understanding of the nature of mathematical thinking, and the way in which that thinking is developed and enhanced by the structures of mathematical thought and language, is, however, unlikely to be either successful or enduring.

A second difficulty lies in the communication of that vision to the classroom teacher, in a way which increases his or her security and confidence, and does not lead to misinterpretation, uncertainty, or confusion of purpose. All too easily a complex vision can become transmuted into a set of methodological mantras which are difficult to dispute because they apparently agree with the vision, although in practice they so distort it as to render it ineffective. In an international US study related to TIMSS, Schmidt *et al*, (1996), came to the conclusions that:

> most teachers do not consciously work through a planning process or routines that order and shape classroom learning activities. Rather, teachers "flow"

> through a familiar activity - teaching in a certain setting - based on their past experiences, training and beliefs. The rules governing their activities, the values guiding their choices, the repertoire of activities and routines from which they choose are largely below the conscious level for most teachers – especially during the course of a lesson,
>
> certain pedagogical strategies are enacted repeatedly in a country's classrooms because they are characteristic of a wide-spread perspective on students, teaching, learning, subject matter content, strategies developed during teacher preparation and training and in the shared professional lives of teachers - in, short, all the factors that interact to determine the typical in a given classroom context (p 71),
>
> how teachers structured discourse around content appeared to vary as a function of how they represented content to themselves and their students (p 72).

The opportunity now exists to recreate the vision for twenty-first century Scotland, a vision which in conception and implementation is neither an adoption or a remake of that of some other country, nor an inward-looking vision which takes no account of comparative studies such as TIMSS. To do so successfully will require jettisoning much of the baggage and slogans of the past; it will require an acknowledgement that the last thirty years have not produced a clear, coherent, communicable, and practically achievable vision for Scottish mathematics education; above all it will require a belief that it needs to be done.

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